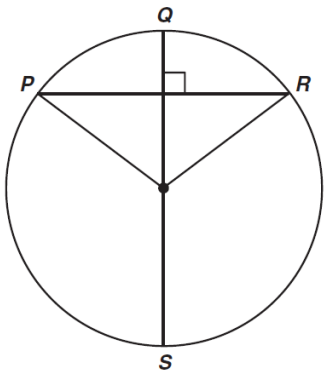
Warm-Up

CST: Geometry 21.0

$\overline{QS}$ is a diameter of the circle below, and $\overline{QS} \perp \overline{PR}$.

If $m\overset{\frown}{PQR} = 106°$, what is $m\overset{\frown}{PS}$?



A 53° C 106°

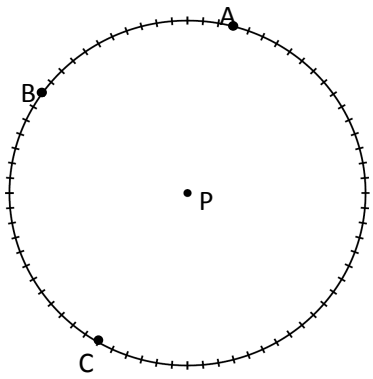
B 74° D 127°

Review: Geometry 21.0

Given Circle P. Find the measure of the following arcs. (Circle P is divided into 5 degree increments)

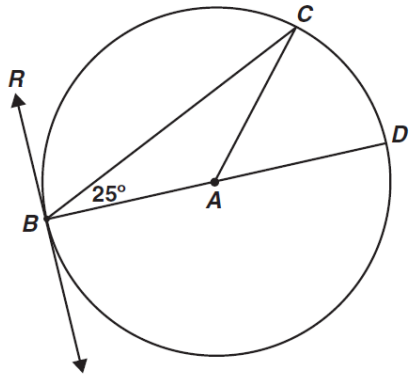
1. $m\overset{\frown}{AB}$

2. $m\overset{\frown}{ACB}$



Current: Geometry 21.0

$\overleftrightarrow{RB}$ is tangent to a circle, whose center is A, at point B. $\overline{BD}$ is a diameter.

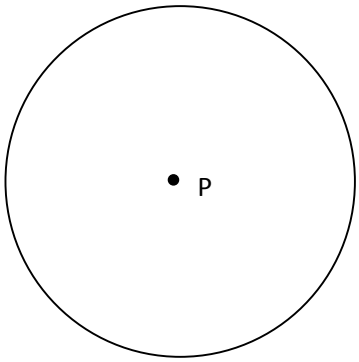


What is $m\angle CBR$?

A 50° C 65°

B 90° D 130°

Other: Geometry 21.0



Given $\odot P$ draw the following.

1. Diameter $\overline{AB}$
2. Tangent $\overleftrightarrow{AC}$
3. Secant $\overleftrightarrow{CD}$

Discovering the Relationship Between Arcs & Angles in Circles

Subject: Geometry

Standard: 21.0

Introduction: Students will discover the properties of central angles & inscribe angles.

Materials:

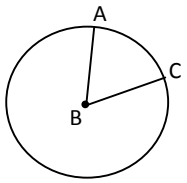
1. Arcs and Angles Chart & Circle Handout
2. Angles: 30°, 60°, and 110° (2 each)
3. 2 Different color highlighters or markers
4. Ruler/Straight edge

Think-Pair-Share

Draw a circle on the overhead & using the **30 degrees angle** ask students to discuss how many unique ways an angle be placed in a circle & intercept at least one arc.

The first angle we are going to investigate is the Central Angle. A central angle is an angle whose vertex lies on the center of a circle.

Given $\odot B$, $\angle ABC$ is a central angle.

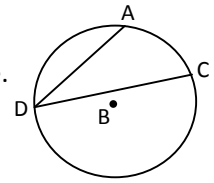


$\angle ABC$ intercepts arc AC

The second angle we are going to investigate is the Inscribed Angle. An inscribed angle is an angle whose vertex lies on the circle.

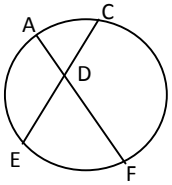
Given $\odot B$, $\angle ADC$ is an inscribed angle.

$\angle ADC$ intercepts arc AC

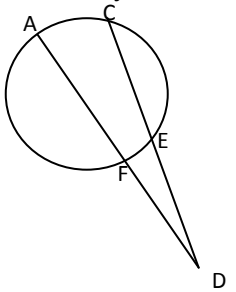


Two Additional Cases

The following angle is formed by two intersecting chords.



The following angle is formed by two secants intersecting.



$\angle ADC$ intercepts two arcs $\overset{\frown}{AC}$ & $\overset{\frown}{EF}$ $\angle ADC$ intercepts two arcs $\overset{\frown}{AC}$ & $\overset{\frown}{EF}$

Hands on Activity

Directions:

Fill in the chart one column at a time. The angles must line up accurately with the tick marks on the circle for accurate results.

Central and Inscribed Angles

- Model for students how they should place the angles on the circle. Have students trace the angle onto the circle. Highlight the intercepted arc & record results.

Special Case: Intersecting Chords/Vertex Inside

- Place the vertices of the two angles with the same measure on point E. The two angles should form a pair of Vertical Angles. Have students trace the angle onto the circle. Highlight the intercepted arcs & record results.

Special Case: Vertex Outside

- Place the vertex of each angle on point D. . Highlight the intercepted arcs & record results.

Ask students to look for patterns within each column. For the Special Case angles, ask students to find the sum or differences between the large and small arcs. Summarize the formula for each case.

Examples

Directions:

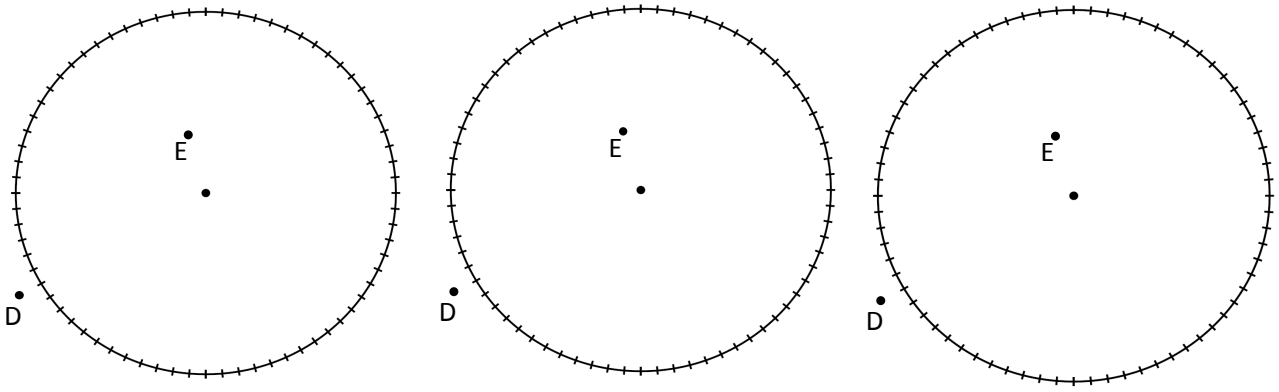
While working through each example ask students, "Where does the vertex of the angle lie?" [inside, on, outside, or on the center]

Have students refer to the filled in chart to find the appropriate formula.

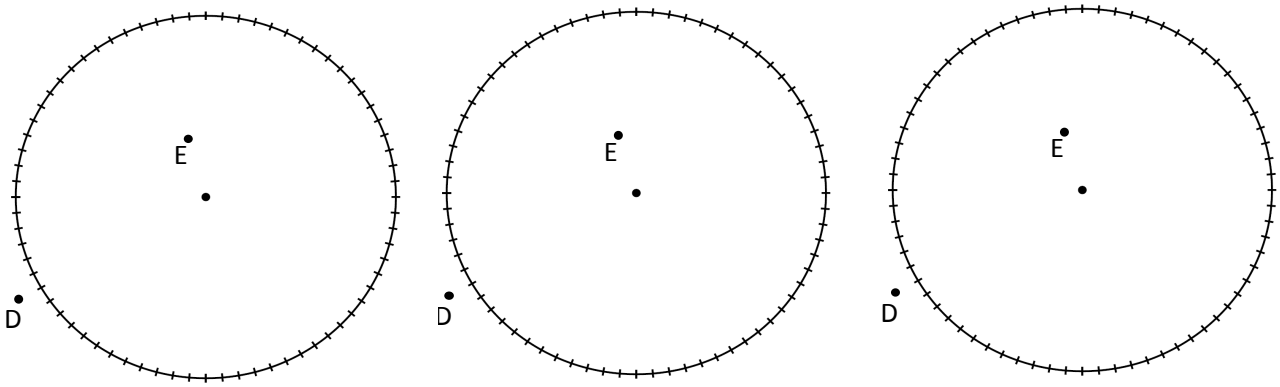
	Measure of Intercepted Arc (degrees)					
	Central Angle	**Inscribed Angle**	**Vertex Inside**		**Vertex Outside**	
			Large Arc	Small Arc	Large Arc	Small Arc
Given Angles						
30°						
60°						
110°						
∠A						

Angles and Arcs

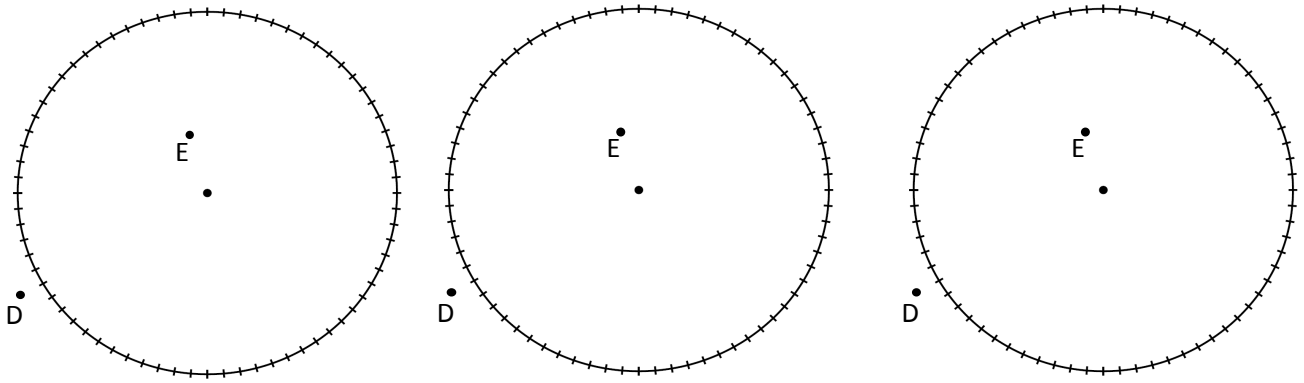
Central



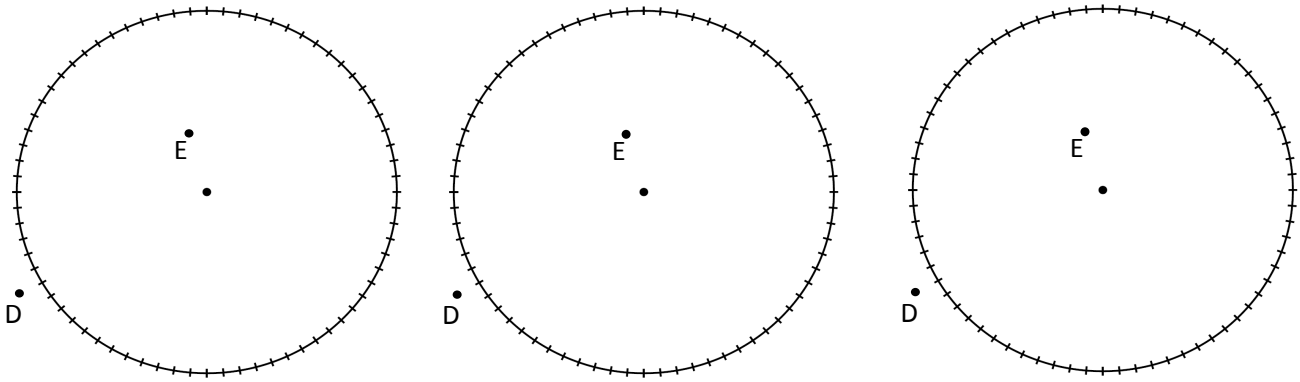
Inscribed

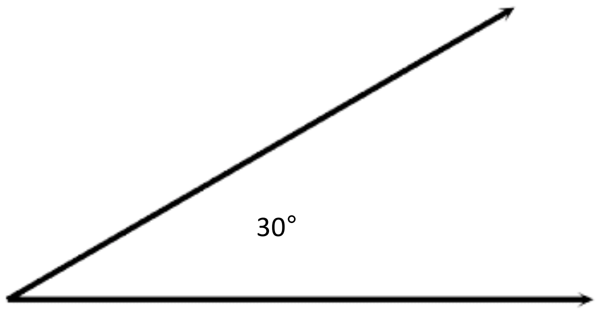


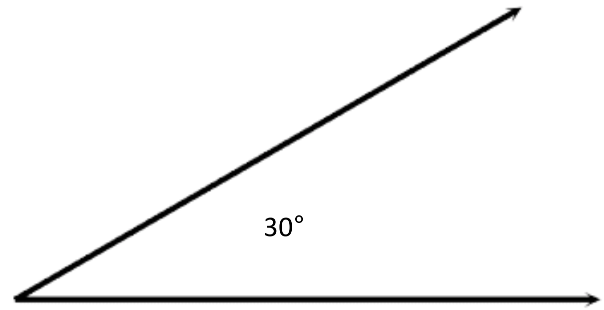
vertex in

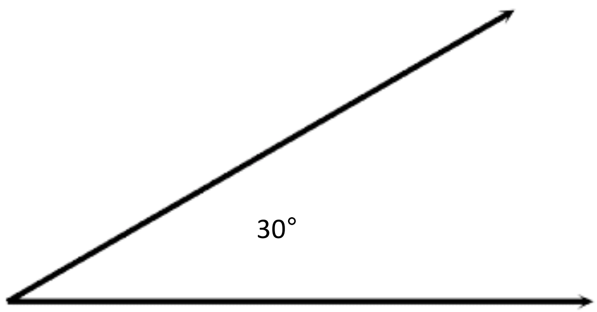


Vertx out




30°


30°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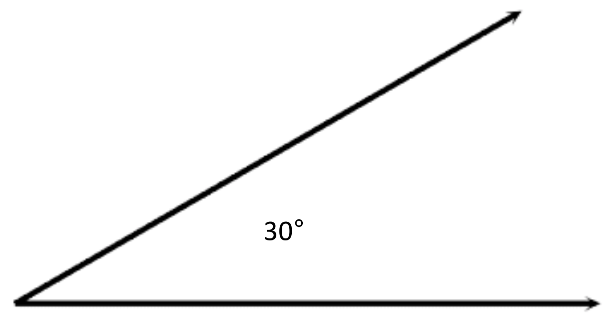
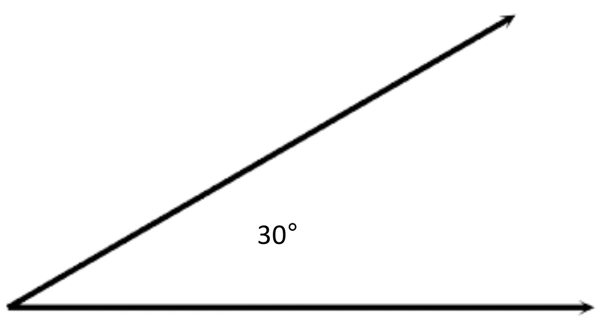
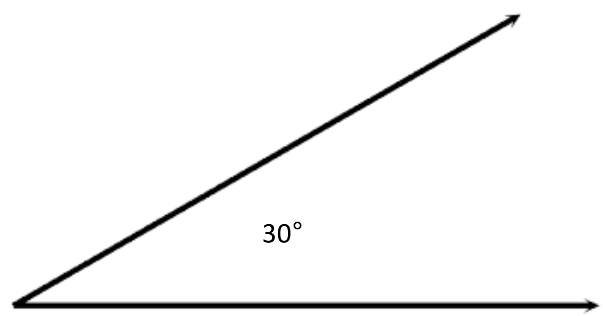
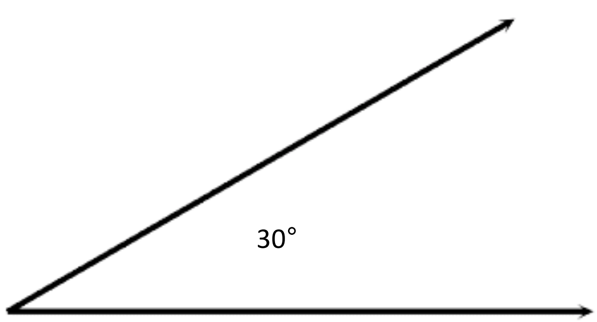
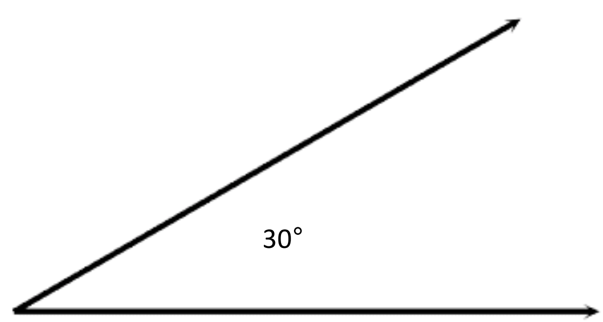
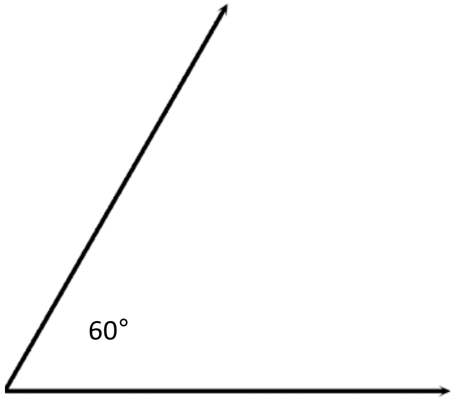

30°

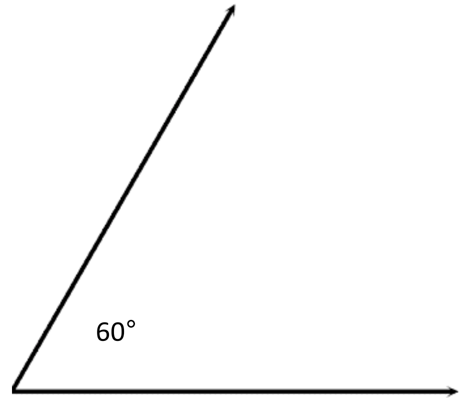

30°

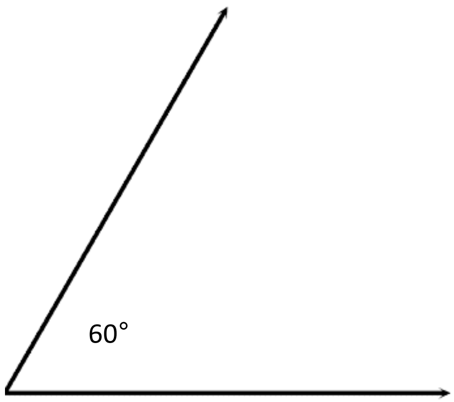

30°

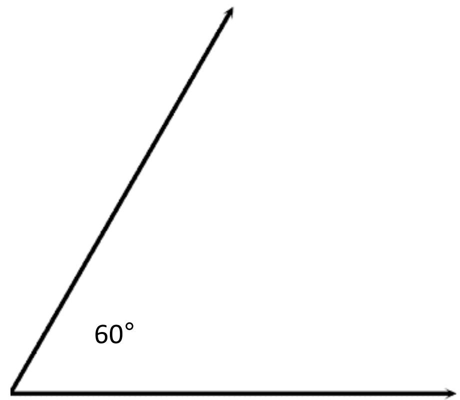

30°

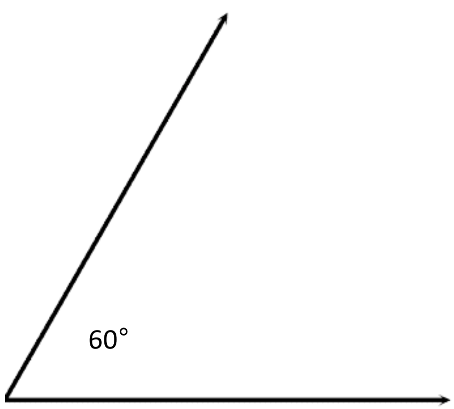

30°


60°


60°


60°


60°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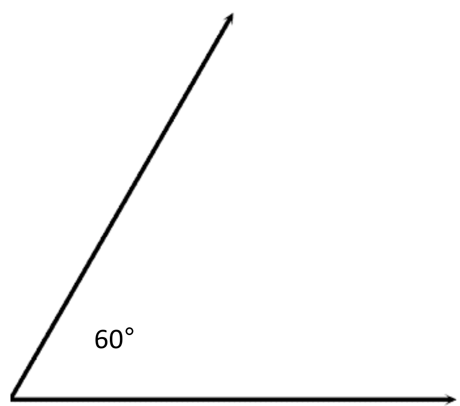
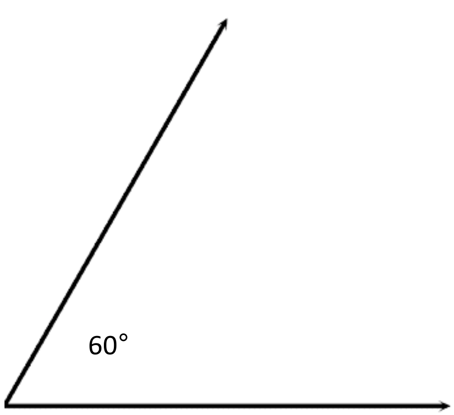
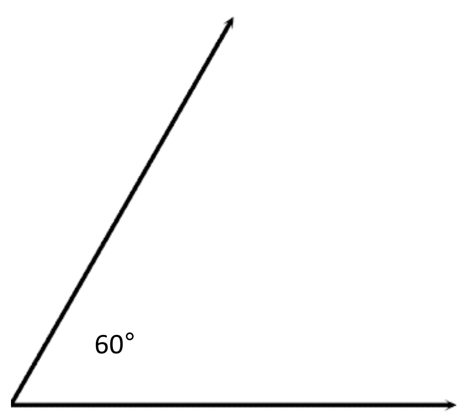
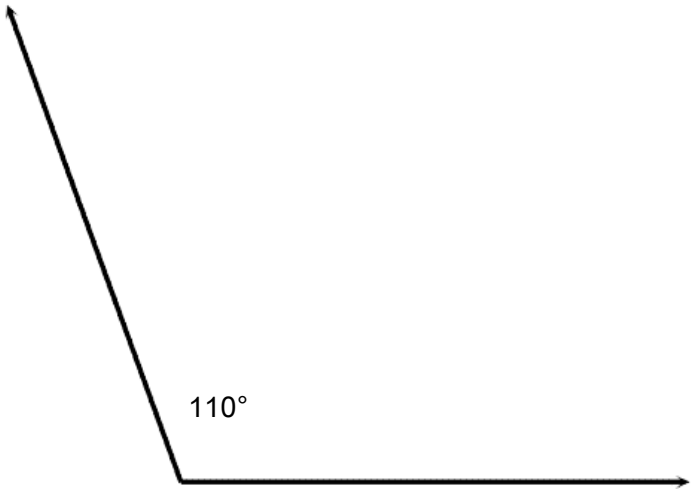
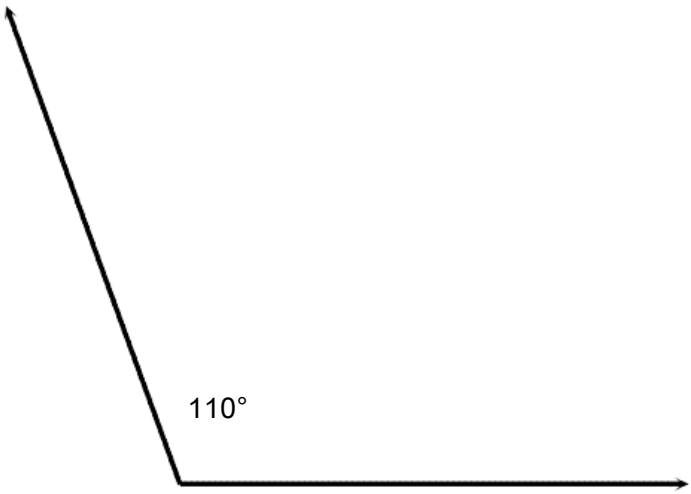
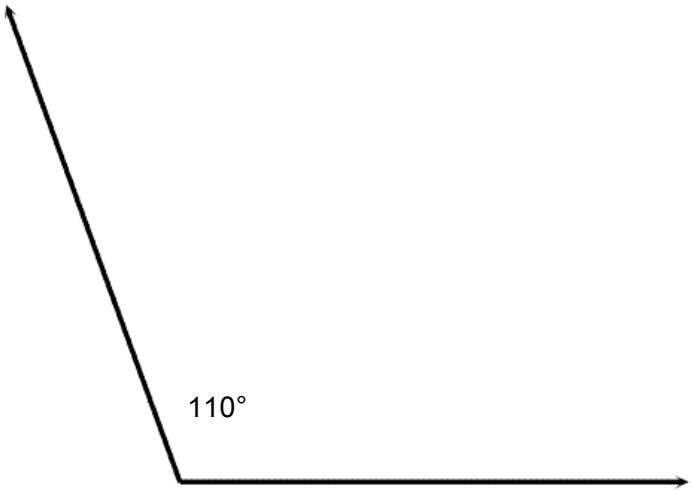

60°

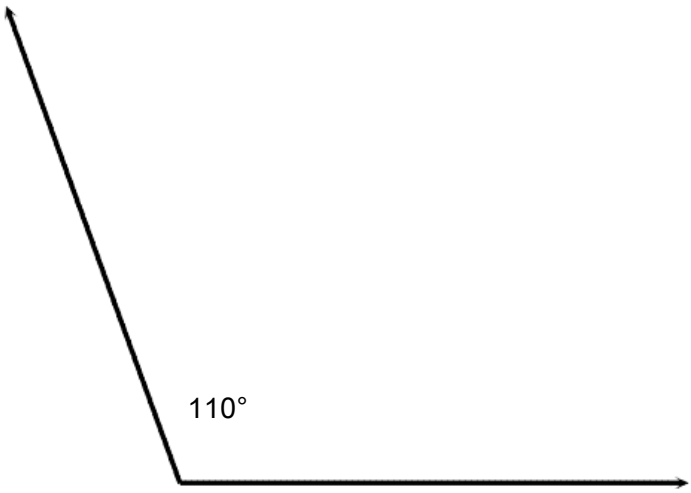

60°


60°


110°


110°


110°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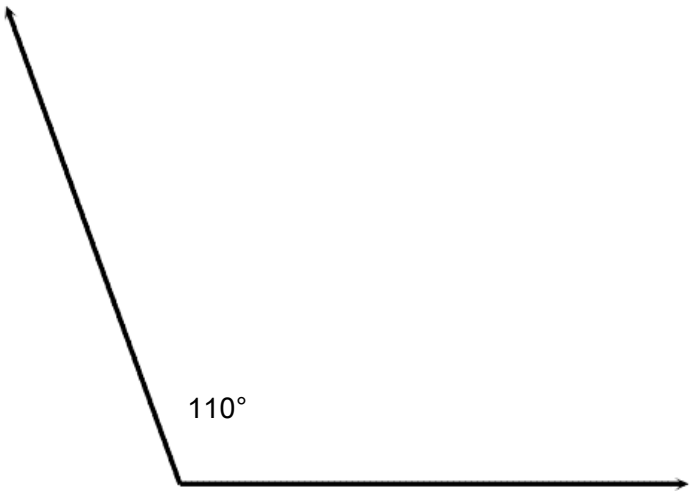
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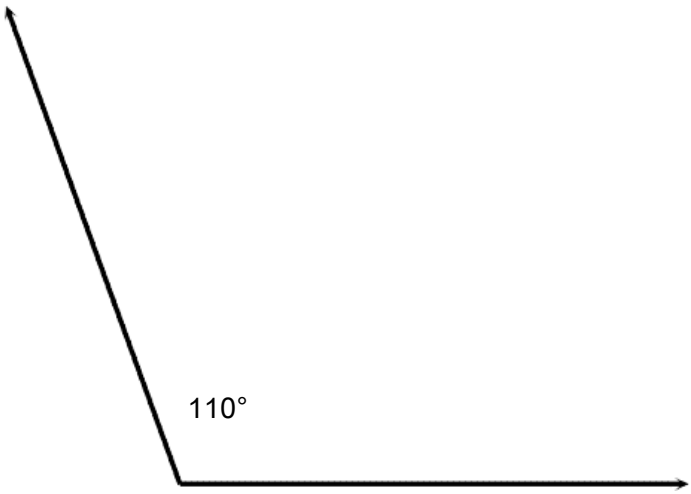
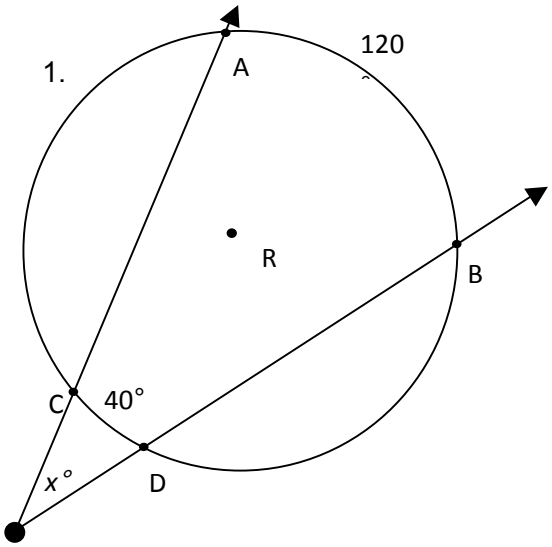
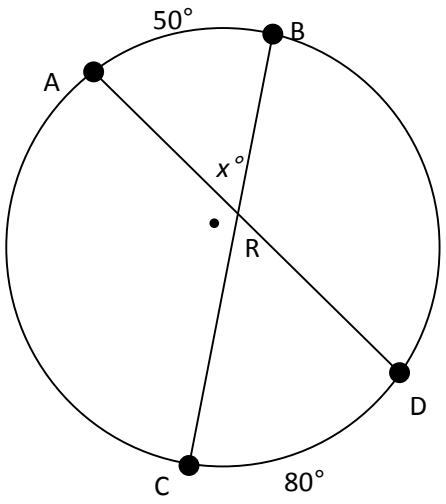

110°

Given Circle R. Find the value of *x*.

1.



2.



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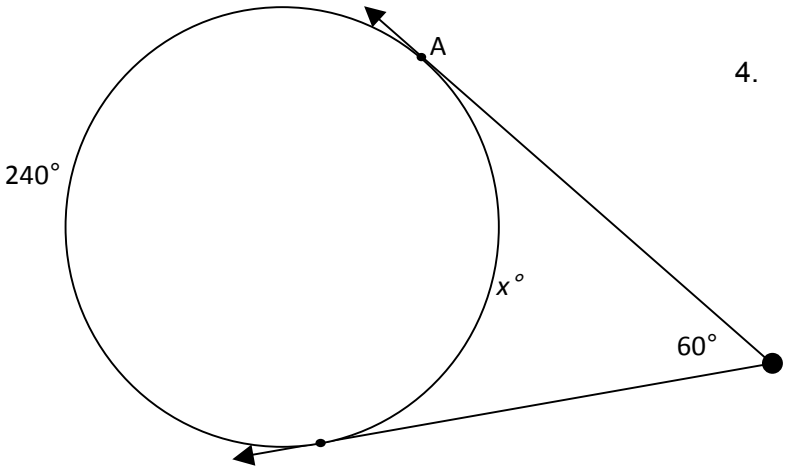

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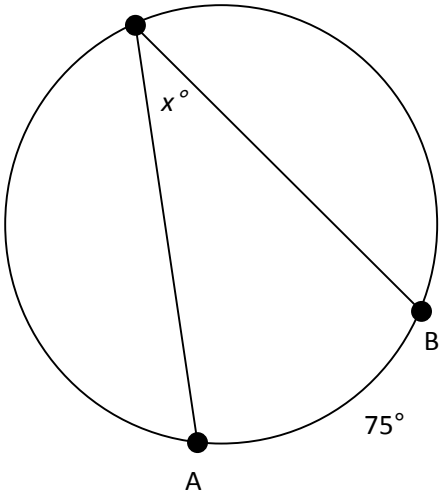
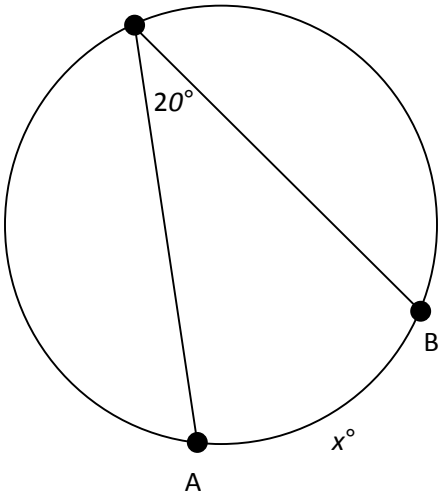
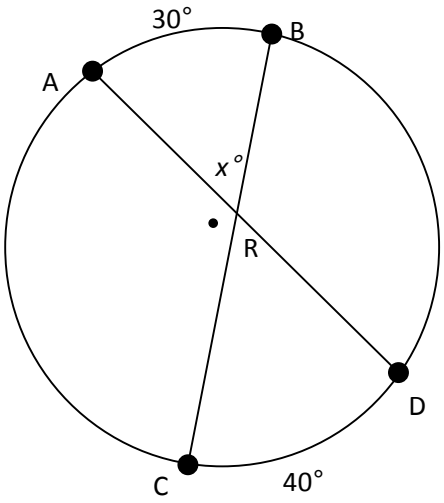


Try This

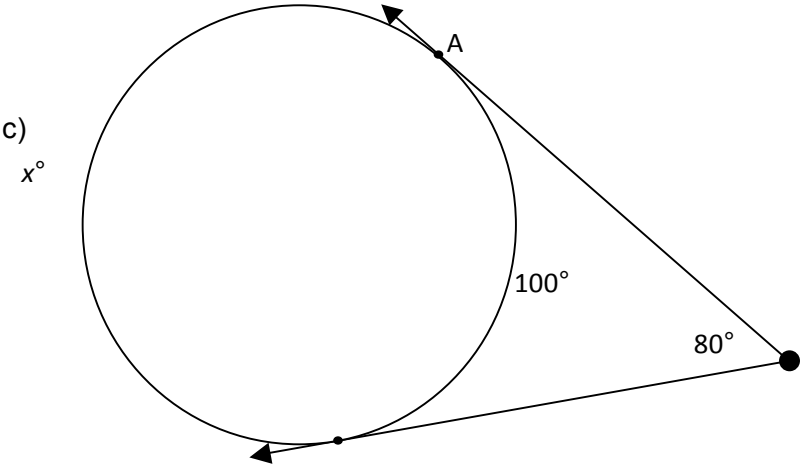
a)



b)



c)



KEY

	Measure of Intercepted Arc (degrees)					
	Central Angle	**Inscribed Angle**	**Vertex Inside****		**Vertex Outside****	
			Large Arc	Small Arc	Large Arc	Small Arc
Given Angles						
30°	30°	60°	35°	25°	65°	5°
60°	60°	120°	75°	45°	135°	15°
110°	110°	220°	120°	100°	250°	30°
∠A	$m\angle A = \text{arc}$	$m\angle A = \frac{1}{2}(\text{arc})$	$m\angle A = \frac{1}{2}(\text{large arc} + \text{small arc})$		$m\angle A = \frac{1}{2}(\text{large arc} - \text{small arc})$	

**Answers will vary

Examples

1.

$$m\angle X = \frac{1}{2}(120-40)$$
$$= \frac{1}{2}(80)$$
$$= 40^o$$

2.

$$m\angle X = \frac{1}{2}(50+80)$$
$$= \frac{1}{2}(130)$$
$$= 65^o$$

3.

$$60 = \frac{1}{2}(240-x)$$
$$2\cdot 60 = 2\cdot\frac{1}{2}(240-x)$$
$$120 = 240-x$$
$$120-240 = 240-240-x$$
$$-120 = -x$$
$$120^o = x$$

4.

$$m\angle X = \frac{1}{2}(75)$$
$$= 37.5^o$$

Your Turn

a)

$$20 = \frac{1}{2}x$$
$$2\cdot 20 = 2\cdot\frac{1}{2}x$$
$$40^o = x$$

b)

$$m\angle X = \frac{1}{2}(30+40)$$
$$= \frac{1}{2}(70)$$
$$= 35^\circ$$

c)

$$80 = \frac{1}{2}(x-100)$$
$$2\cdot 80 = 2\cdot\frac{1}{2}(x-100)$$
$$160 = x-100$$
$$160+100 = x-100+100$$
$$260^\circ = x$$